Chapter 13: Applications

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Examples:

a) For which $k \in \mathbb{R}$ is (-4,5,6) $\in span\{(-2,1,-1),(4,1,4),(-8,13,k)\}$?

This means:
$(-4,5,6) = a(-2,1,-1) + b(4,1,4) + c(-8,13,k)$ for some $a,b,c \in \mathbb{R}$

translate into a linear system

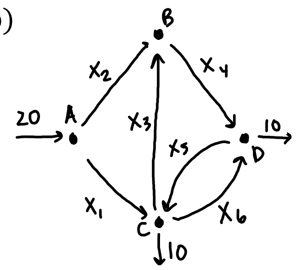
$$\left[\begin{array}{ccc|c} -2 & 4 & -8 & -4 \\ 1 & 1 & 13 & 5 \\ -1 & 4 & k & 6 \end{array}\right] \underset{\sim}{-\tfrac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 1 & 1 & 13 & 5 \\ -1 & 4 & k & 6 \end{array}\right]$$

$$\underset{\sim}{\begin{matrix} R_2 - R_1 \to R_2 \\ R_3 + R_1 \to R_3 \end{matrix}} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 3 & 9 & 3 \\ 0 & 2 & k+4 & 8 \end{array}\right] \underset{\sim}{\tfrac{1}{3}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 2 & k+4 & 8 \end{array}\right]$$

$$R_3 - 2R_2 \to R_3 \left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & k-2 & 6 \end{array}\right]$$

If $k = 2$: no solution!
If $k \neq 2$: $(-4,5,6) \in \text{span}\{(-2,1,-1),(4,1,4),(-8,13,\text{k})\}$

b)



flow in = flow out

A: $20 = x_1 + x_3$

B: $x_2 + x_3 = x_4$

C: $x_1 + x_5 = 10 + x_3 + x_6$

D: $x_4 + x_6 = 10 + x_5$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 0 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 1 & -1 & 1 & 10 \end{array}\right] \sim \; \sim \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \end{matrix} \\ \left[\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & 1 & -1 & 10 \\ 0 & 1 & 1 & 0 & -1 & 1 & 10 \\ 0 & 0 & 0 & 1 & -1 & 1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{matrix} & & r & & s & t & \end{matrix} \end{array} \text{RREF}$$

$x_1, \dots, x_6 \in \mathbb{N}$

$$S = \left\{ \begin{pmatrix} 10+r-s+t \\ 10-r+s-t \\ r \\ 10+s-t \\ s \\ t \end{pmatrix} \middle| \; r,s,t \in \mathbb{R} \right\}$$

Now, if $\overline{CD}$ is closed in both direction, what is the maximum flow along AC. CD is both x_5 and x_6, so s=t=0. Then we have:

$\Big(\big(10+r$

Now, if CD is closed in both direction, what is the maximum flow along AC. CD is both x_5 and x_6, so s=t=0. Then we have:

$$S = \left\{ \begin{pmatrix} 10+r \\ 10-r \\ r \\ 10 \\ 0 \\ 0 \end{pmatrix} \middle| r \in \mathbb{R} \right\}$$

If we analyze our constraints, we obtain:

$x_1 \geq 0 \qquad x_1 = 10+r \qquad r \geq 0$

$x_2 \geq 0 \qquad x_2 = 10-r \qquad 0 \leq r \leq 10$

$x_3 \geq 0 \qquad x_3 = r \qquad r \geq 0$

$x_4 \geq 0 \qquad x_4 = 10$

$x_5 \geq 0 \qquad x_5 = 0$

$x_6 \geq 0 \qquad x_6 = 0$

$0 \leq r \leq 10$

For AC to be maximum, r must be maximum.
$AC = x_1 = 10 + r$
$= 10 + 10 = 20$
Therefore, the maximum flow along AC is 20 cars.